22147406

## MATHEMATICAL STUDIES

STANDARD LEVEL

## PAPER 2

Wednesday 14 May 2014 (morning)
1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the Mathematical Studies SL formula booklet is required for this paper.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is [90 marks].

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 16]

Tomek is attending a conference in Singapore. He has both trousers and shorts to wear. He also has the choice of wearing a tie or not.

The probability Tomek wears trousers is 0.3 . If he wears trousers, the probability that he wears a tie is 0.8 .

If Tomek wears shorts, the probability that he wears a tie is 0.15 .
The following tree diagram shows the probabilities for Tomek's clothing options at the conference.

(a) Find the value of
(i) A ;
(ii) B ;
(iii) C .
(b) Calculate the probability that Tomek wears
(i) shorts and no tie;
(ii) no tie;
(iii) shorts given that he is not wearing a tie.

## (Question 1 continued)

The conference lasts for two days.
(c) Calculate the probability that Tomek wears trousers on both days.
(d) Calculate the probability that Tomek wears trousers on one of the days, and shorts on the other day.

Please do not write on this page.
Answers written on this page will not be marked.
2. [Maximum mark: 14]

A cross-country running course consists of a beach section and a forest section. Competitors run from A to B , then from B to C and from C back to A .

The running course from $A$ to $B$ is along the beach, while the course from $B$, through C and back to A , is through the forest.

The course is shown on the following diagram.


Angle ABC is $110^{\circ}$.
It takes Sarah 5 minutes and 20 seconds to run from A to B at a speed of $3.8 \mathrm{~m} \mathrm{~s}^{-1}$.
(a) Using 'distance $=$ speed $\times$ time', show that the distance from A to B is 1220 metres correct to 3 significant figures.

The distance from B to C is 850 metres. Running this part of the course takes Sarah 5 minutes and 3 seconds.
(b) Calculate the speed, in $\mathrm{ms}^{-1}$, that Sarah runs from B to C.
(c) Calculate the distance, in metres, from $\mathbf{C}$ to $\mathbf{A}$.
(d) Calculate the total distance, in metres, of the cross-country running course.
(e) Find the size of angle BCA.
(f) Calculate the area of the cross-country course bounded by the lines AB , BC and CA .
3. [Maximum mark: 10]

A survey was conducted to determine the length of time, $t$, in minutes, people took to drink their coffee in a café. The information is shown in the following grouped frequency table.

| Time, $t$ <br> (minutes) | Number of <br> People |
| :---: | :---: |
| $0<t \leq 5$ | 3 |
| $5<t \leq 10$ | 5 |
| $10<t \leq 15$ | 12 |
| $15<t \leq 20$ | 14 |
| $20<t \leq 25$ | 16 |
| $25<t \leq 30$ | 10 |

(a) Write down the total number of people who were surveyed.
(b) Write down the mid-interval value for the $10<t \leq 15$ group.
(c) Find an estimate of the mean time people took to drink their coffee.

The information above has been rewritten as a cumulative frequency table.

| Time, $t$ <br> (minutes) | $t \leq 5$ | $t \leq 10$ | $t \leq 15$ | $t \leq 20$ | $t \leq 25$ | $t \leq 30$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative <br> frequency | 3 | 8 | 20 | $a$ | 50 | $b$ |

(d) Write down the value of $a$ and the value of $b$.

## (Question 3 continued)

This information is shown in the following cumulative frequency graph.

(e) For the people who were surveyed, use the graph to estimate
(i) the time taken for the first 40 people to drink their coffee;
(ii) the number of people who take less than 8 minutes to drink their coffee;
(iii) the number of people who take more than 23 minutes to drink their coffee.
4. [Maximum mark: 19]

## Give your answers to parts (a) to (e) to the nearest dollar.

On Hugh's 18th birthday his parents gave him options of how he might receive his monthly allowance for the next two years.

Option A \$60 each month for two years
Option B $\$ 10$ in the first month, $\$ 15$ in the second month, $\$ 20$ in the third month, increasing by $\$ 5$ each month for two years

Option C $\$ 15$ in the first month and increasing by $10 \%$ each month for two years

Option D Investing $\$ 1500$ at a bank at the beginning of the first year, with an interest rate of $6 \%$ per annum, compounded monthly.

Hugh does not spend any of his allowance during the two year period.
(a) If Hugh chooses Option A, calculate the total value of his allowance at the end of the two year period.
(b) If Hugh chooses Option B, calculate
(i) the amount of money he will receive in the 17th month;
(ii) the total value of his allowance at the end of the two year period.
(c) If Hugh chooses Option C, calculate
(i) the amount of money Hugh would receive in the 13th month;
(ii) the total value of his allowance at the end of the two year period.
(d) If Hugh chooses Option D, calculate the total value of his allowance at the end of the two year period.
(e) State which of the options, A, B, C or D, Hugh should choose to give him the greatest total value of his allowance at the end of the two year period.

Another bank guarantees Hugh an amount of $\$ 1750$ after two years of investment if he invests $\$ 1500$ at this bank. The interest is compounded annually.
(f) Calculate the interest rate per annum offered by the bank.
5. [Maximum mark: 17]

A parcel is in the shape of a rectangular prism, as shown in the diagram. It has a length $l \mathrm{~cm}$, width $w \mathrm{~cm}$ and height of 20 cm .

The total volume of the parcel is $3000 \mathrm{~cm}^{3}$.
(a) Express the volume of the parcel in terms of $l$ and $w$.
(b) Show that $l=\frac{150}{w}$.

The parcel is tied up using a length of string that fits exactly around the parcel, as shown in the following diagram.

(c) Show that the length of string, $S \mathrm{~cm}$, required to tie up the parcel can be written as

$$
\begin{equation*}
S=40+4 w+\frac{300}{w}, 0<w \leq 20 . \tag{2}
\end{equation*}
$$

(d) Draw the graph of $S$ for $0<w \leq 20$ and $0<S \leq 500$, clearly showing the local minimum point. Use a scale of 2 cm to represent 5 units on the horizontal axis $w(\mathrm{~cm})$, and a scale of 2 cm to represent 100 units on the vertical axis $S(\mathrm{~cm})$.
(e) Find $\frac{\mathrm{d} S}{\mathrm{~d} w}$.
(f) Find the value of $w$ for which $S$ is a minimum.
(g) Write down the value, $l$, of the parcel for which the length of string is a minimum.
(h) Find the minimum length of string required to tie up the parcel.
6. [Maximum mark: 14]

The front view of the edge of a water tank is drawn on a set of axes shown below. The edge is modelled by $y=a x^{2}+c$.


Point P has coordinates $(-3,1.8)$, point O has coordinates $(0,0)$ and point Q has coordinates $(3,1.8)$.
(a) Write down the value of $c$.
(b) Find the value of $a$.
(c) Hence write down the equation of the quadratic function which models the edge of the water tank.

## (Question 6 continued)

The water tank is shown below. It is partially filled with water.

(d) Calculate the value of $y$ when $x=2.4 \mathrm{~m}$.
(e) State what the value of $x$ and the value of $y$ represent for this water tank.
(f) Find the value of $x$ when the height of water in the tank is 0.9 m .

The water tank has a length of 5 m .
(g) When the water tank is filled to a height of 0.9 m , the front cross-sectional area of the water is $2.55 \mathrm{~m}^{2}$.
(i) Calculate the volume of water in the tank.

The total volume of the tank is $36 \mathrm{~m}^{3}$.
(ii) Calculate the percentage of water in the tank.

